

# Constructal View of Formation of Dissimilar Patterns inside Similar Living Systems

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## Abstract

This paper proposes to apply constructal theory to explain the differences found in morphology of similar living systems. Our study shows that the shape of the biological system is influenced by the environmental parameters. The shape of the biological system is optimized to it provide the internal paths such maximizes nutrients access with time. An optimal shape (architecture) emerges for survival.

## Constructal Law and the Emergence of Shape and Structure in Systems

The constructal law was stated by Adrian Bejan in 1996 as follows: ““for a finite system to persist in time, it must evolve in such a way that it provides access to the currents that flow through it” (Bejan, 2000). The constructal law can be applied for any dissipative systems, whether they are living or non-living, and constitutes a major step toward making system design a science. The geometry of the system is free to change while its global performance is being maximized. The morphing of structure is the result of the conflict between the global objective and the global constraints. In other words, objectives and constraints together with physics laws are fingers of the same hand that give shape to the final system. There is a growing body of evidence that constructal theory could be applied to explain and predict the shape (architecture) of living systems (Bejan, 2000; Reis et al, 2004; Bejan & Marden, 2006; Miguel, 2006). The objective of this study is to propose an answer for the following questions: Why is there a formation of dissimilar patterns inside similar systems? Is there an optimal shape for survival in nature? The formation of stony corals and bacterial colonies are the examples used in this paper.

## Nutrient Transport Flow and Growth Model for Biological Systems

Nutrient transport within a fluid is an example of a transport that can be analyzed within the framework of a convective–dispersive phenomenon (Miguel, 2006). The time-scales can be obtained by applying scale analysis to the equation that describes this phenomenon. The characteristics times corresponding to the dispersive and convective driven transport are (Miguel, 2006)

$$\tau_d \sim \frac{(1+\kappa)L^2}{D}; \tau_c \sim \frac{(1+\kappa)L}{u} \quad (1)$$

Here  $\kappa$  is the dimensionless nutrients depletion,  $D$  is the nutrient diffusion coefficient,  $u$  is the fluid velocity,  $L$  is the length scale of nutrients propagation, and  $\tau_d$  and  $\tau_c$  are the characteristic time of nutrient dispersive and convective driven transport, respectively. The transition time ( $\tau_t$ ) from dispersive to convective driven transport is obtained from

the interception of  $\tau_d$  and  $\tau_c$ , being  $\tau_t \sim (1+\kappa)D/u^2$ . If  $\tau < \tau_t$ , then dispersion of nutrients overcomes convection but when  $\tau > \tau_t$  nutrient transport is mainly driven by convection. For the sake of parsimony, we assume that the growth process is characterized by a constant rate but different growth models can be adopted.

### The Emergence of Shape and the Nutrient Flow

Both stony corals and bacterial colonies develop branched or more circular shapes, apparently due to the variability of environmental parameters (see for example, Kaandorp & Sloat, 2001; Thar & Kuhl, 2005). These geometries are different regarding their ability to fill the space, and consequently to extract nutrients from the environment in the shortest time. It is easy to prove that that the circular shape is the most effective arrangement for filling the space in the shortest time (Miguel, 2006). Why should a branched/needle shape geometry occurs, even if it is not the most effective arrangement? Consider, for instance, stony corals or bacteria growing in sheltered places and within delimited environments (Petri dishes), respectively. In this situation, transition from dispersive to convective driven transport is not very likely to occur and the transport of nutrients is linked to a dispersive phenomenon.

The biological system starts to grow at its birth. Immediately after, nutrients close to the biological system are quickly depleted. This consumption of nutrients in the surroundings of the system causes a decrease of nutrient concentration which triggers a diffusive wave of nutrients defined by the characteristic length, given by Eq. (1), which the speed of propagation is  $v_d = dL/d\tau \sim [D/(1+\kappa)\tau]^{1/2}$ . The initial speed of nutrient propagation (for  $\tau=0$ ) is much larger than the growth speed of the biological system but decreases with the inverse of the square root of the time. As a result, in time the speed of nutrient propagation ends up falling behind the growth speed of the system (Fig. 1).

Figure 1: Nutrient propagation, biological system growth and the occurrence of critical times

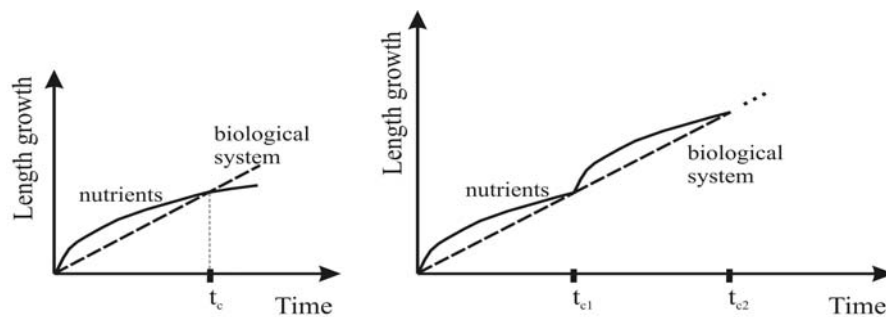


Fig. 1 shows that a critical time ( $\tau_c$ ) is reached when the characteristic length of the biological system overtakes the characteristic length corresponding to the nutrient dispersive driven transport. Despite the circular shape is the most effective shape to extract nutrients from the environment in the shortest time, at times slightly larger than the critical time, the biological system starts to grow outside of the nutrient diffusion region. To survive, a needle shape is then generated to promote the easiest access to the nutrients. This “biological channeling” enables the biological system to experience again growth inside the nutrient diffusion region from  $t_{c1}$  up to  $t_{c2}$  (Fig. 1). At times slightly greater than  $t_{c2}$ , the needle length sticks once again its tip out the nutrient diffusion

region. Therefore, new needles are “generated” in order to experience the growth inside the nutrient diffusion region until a new critical time is reached. Each needle generates a new group of needles, and the result of this scenario is a system having a dendritic shape because this is the more competitive shape configuration. The size of the nutrient particles and ambient temperature also affect the shape of the biological system because they affect the nutrient diffusion coefficient (Stokes–Einstein equation). Temperature and nutrient size have dissimilar effects on the diffusion coefficient. An increase of the nutrient size leads to a decrease in the diffusion coefficient. As a result, the biological system morphs into a more branched shape with needles being created to provide an easier access to nutrients. The ambient temperature affects the diffusion coefficient directly, but also through viscosity (liquids become less viscous with the temperature). An increase of the temperature leads to a longer critical time and less levels of branching. Consider now the case of biological systems growing in open sites where convection currents are important. The fluid velocity surrounding the system is usually much larger than the growth speed of the biological system (for instance, the growing speed of stony corals is few millimeters per year). Therefore, the length of the biological system never overtakes the characteristic length corresponding to the nutrient driven transport. The critical time is never reached and the system grows always inside a region where nutrients are readily available. Consequently, the system is able to develop the most effective shape for filling the space in the shortest time – the round and compact shape.

### **Final Remarks**

The constructal law is applied in this study to explain the formation of dissimilar shapes inside similar systems. The results presented here agree with previous studies (Kaandorp & Sloat, 2001; Thar & Kuhl, 2005). We showed that in order to persist in time, these systems must evolve in such a way that an easy access to nutrients is ensured. Their shapes (architecture) do not develop by chance; they represent the optimum structure serving their ultimate purpose – the survival.

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